Linear Algebra Tutorial

RREF of A

```
A = matrix([[3,4,7,-1],[2,6,8,-4],[-5,3,-2,-8],[7,-2,5,9]])
show(A.rref())
```

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

RREF of (A|b)

```
A = matrix([[1,2,3],[4,5,6],[7,8,2]])
b = vector([-1,4,-7])
Aaug = A.augment(b); show(Aaug.rref())
```

$$\begin{pmatrix} 1 & 0 & 0 & \frac{139}{21} \\ 0 & 1 & 0 & -\frac{152}{21} \\ 0 & 0 & 1 & \frac{16}{7} \end{pmatrix}$$

Solve Ax=b (note that solve_right does NOT necessarily give all solutions)

```
A = matrix([[3,4,-7,2],[2,6,9,-2],[-5,3,2,-13],[7,-2,5,16]])
b = vector([5,27,11,-1])
A.solve_right(b)
```

```
(0, 3, 1, 0)
```

```
A = matrix([[3,4,-7,2],[2,6,9,-2],[-5,3,2,-13],[7,-2,5,16]])
b = vector([5,27,11,-1])
Aaug = A.augment(b); show(Aaug.rref())   # same system, infinite number
of solutions
```

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Determinant of A

```
A = matrix([[-3,4,8,-2],[2,6,8,-4],[-5,-9,-2,-8],[7,-2,5,9]])
show(A.determinant())
```

$$-6624$$

Eigenvalues/vectors

```
A = matrix([[0,1,0,0],[0,0,1,0],[0,0,0,1],[-9,-6,3,0]])
show(A.eigenvalues())
```

$$[-1.919039750549684?, -1.132826740049513?, 1.525933245299598? - 1.3459113$$

```
A = matrix([[-1,1/80,1/16],[1/2,-961/80,0],[1/2,12,-1/16]])
A.eigenvectors_right()
```

```
[(0, [
  (1, 40/961, 15368/961)
  ], 1),
 (-12.010216418744899?, [(1, 218.9543283748980?,
-219.9543283748980?)], 1),
 (-1.064783581255101?, [(1, 0.04567162510200666?,
-1.045671625102007?)], 1)]
```

QR factorization

```
A = matrix(CDF,[[1,2,3],[4,5,6],[7,8,2]])   # option CDF forces numerical
computation
show(A.QR())
```

$$\left(\begin{pmatrix} 0.123091490979 & 0.904534033733 & -0.408248290464 \\ 0.492365963917 & 0.301511344578 & 0.816496580928 \\ 0.861640436855 & -0.301511344578 & -0.408248290464 \end{pmatrix}, \begin{pmatrix} 8.1240384046 \\ 0.0 \\ 0.0 \end{pmatrix}\right.$$

SVD

```
A = matrix(CDF,[[1,2,3],[4,5,6],[7,8,2]])   # option CDF forces numerical
computation
show(A.SVD())  # first matrix is U, middle matrix is singular values,
last matrix is V^H
```

$$\left(\begin{pmatrix} -0.235655948374 & -0.431176804405 & -0.870949388506 \\ -0.605866578612 & -0.635534174798 & 0.478562432274 \\ -0.759863121171 & 0.640455209992 & -0.111468206575 \end{pmatrix}, \begin{pmatrix} 13.77609060 \\ 0 \\ 0 \end{pmatrix}\right.$$

Plot function of one variable

```
var("x")
f(x) = (x^2*cos(2*x)-1)*exp(-x)
g(x) = x^3/(1+x^6)
plotf = plot(f(x), (x,0,12), color='blue', linestyle='-', thickness=1)
show(plotf, figsize=[5,3])
# plotg = plot(g(x), (x,0,12), color='red', linestyle=':', thickness=1)
# plotall = plotf + plotg
# show(plotall, figsize=[5,3])
```